Buoyancy is defined as the tendency of a fluid to exert a supporting upward force on a body placed in a fluid (i.e., a liquid or a gas). The fluid can be a liquid, as in the case of a boat floating on a lake, or the fluid can be a gas, as in a helium-filled balloon floating in the atmosphere. An elementary application of buoyancy can be seen when trying to push an empty water bottle downwards in a sink full of water. When applying a downward force to the water bottle from your hand, the water bottle will stay suspended in place. But, as soon as you remove your hand from the water bottle, the water bottle will float to the surface. The buoyant force on the object determines whether or not a given object will sink or float in a fluid.

It can be determined if an underground concrete structure will float or sink using basic principles and applying this to a concrete structure. Essentially a concrete structure will not float if the sum of the vertical downward forces (gravitational, $W$) is greater than the vertical upward force (buoyant, $F_b$). When applying this principle to a structure below grade, it can be said that; if the buoyant force ($F_b$) is greater than the mass of the structure and the combined mass of soil surcharges and objects contained within the structure, the structure will float.

People have been aware of objects floating or sinking in a fluid since the beginning of time. It wasn’t officially documented and conceptually grasped until Archimedes (287-212 B.C.), established the theory of floatation and defined the buoyancy principle. Apparently (while in his bath) Archimedes realized that submerged objects always displace fluid upward (water in the bath rose while he was submerged). Then with that observation, he concluded that this force (buoyant) must be equal to the weight of the displaced fluid. Archimedes then went on to state that a solid object would float if the density of the solid object was less than the density of the fluid and vice versa.
Section 2- Determining Water Table Levels

2.1 Review Plans, Specifications, Soils Reports and Boring Logs

When designing an underground precast concrete structure, it is necessary to know what structure to make as well as what its intended use is. Typically a contractor who is in need of a precast structure will come to precasters with details on what they need and give design requirements and information on the underground conditions. But, not always do they inform precasters about every detail, especially job site conditions and problems in the construction area. Job site conditions and underground conditions are vital pieces of information needed for the design calculations to optimize the performance of the structure in the installed condition and to prevent floatation. So, how does the design engineer determine when there could be a potential problem with the jobsite conditions and with floatation?

First off, the design engineer should do his/her research to review and investigate the plans, specifications and soils reports to gain more insight about the project and the underground conditions. After obtaining the design requirements and specifications for the structural design the design engineer should obtain extensive information on the soils and underground conditions. One of the first factors that must be determined when analyzing an area, in which the concrete structure will be placed below grade, is the water table, ground water level. Obtaining this information will help the designers identify areas where flotation could potentially be a problem and areas where flotation will most
likely not be a factor in the design. So, how can one determine the water table level in the project area?

The design engineer should check the soils report to obtain more information on the area. The soils report is most likely the most reliable source of information since it’s based on a study of the jobsites’ conditions. If there isn’t a soils report, core drilling may be necessary. By core drilling in the vicinity of the project, the depth of the water level from grade can be determined. It should be noted that ground water levels identified on boring reports are only a snap shot and may not account for seasonal variations. Another possible source of information would be from local well drillers. They should maintain records of water table levels in areas which they drill. If there isn’t any of the above information available, you may want to ask your local excavation contractors, since they have firsthand experience with ground water in the area. If you’re out of options and you can’t find out any of the above information, it is recommended to consider a design for water levels at grade, even if flooding in that area is not common. This is considered a conservative approach by many design engineers. A conservative design approach may offset unnecessary and unforeseen cost when sufficient information about the soil/site conditions is unavailable.

After the water table/level has been determined and it is known that there will most likely not be a problem with buoyancy or flotation issues, the designer can focus in on maximizing the structure without buoyant forces being exerted on the structure. In most cases flotation will not be a problem in areas of the country without groundwater (parts of Texas, Arizona, Nevada, etc.), and where the ground water is below the anticipated depth of the structure. The fact that the buoyancy force ($F_b$) exists presupposes that the water table at the site is believed to be some distance above the lowest point of the installed structure. But, if your structure is to be placed above the ground water level (according to the sites’ water table), less concern is needed. On the other hand, areas where flotation causes potential problems are regions where the water level is at grade (valleys, ocean shores), and areas where ground water is present below grade, at time of installation (before soil has had a chance to become a solid mass).

2.2 Be Aware of Seasonal and Regional Variations

The water table is the upper level of an underground surface in which the soil is saturated with water. The water table fluctuates both with the seasons and from year to year because it is affected by climatic variations and by the amount of precipitation used by vegetation. It also is affected by withdrawing excessive amounts of water from wells or by recharging them artificially (5). The design engineer should make certain to factor the water table level seasonal and regional fluctuations into the design of an underground precast concrete structure. This will ensure that the underground structure will not float or shift upwards from a water table level miscalculation.
2.3 Err on the Conservative Side

If there are no soils reports or previous information available on the water table levels and fluctuations (seasonal and regional), most design engineers will design the structure on the conservative side. This will ensure that the structure will be able to withstand seasonal and regional fluctuations.

Designing on the conservative side refers to designing a structure with the water level at grade, even if flooding in that area is not common. A conservative design approach may contribute to offsetting unnecessary and unforeseen cost when sufficient information about the soil/site conditions is unavailable. Therefore, overdesigning a structure should be kept to a minimum since this would add substantial costs to production.
Section 3- Computing Downward (Gravity) Forces

After the water table level has been determined, the design engineer needs to look at computing all the downward forces that will be acting on the structure. All vertical downward forces are caused by gravitational effects, which need to be calculated in the design of an underground structure in order to determine if the total downward forces (gravitational, \( W_T \)) are greater than the upward force (buoyant \( F_B \)). The total downward force (\( W_T \)) is calculated by the summation of all downward vertical forces (\( W \)).

\[
W_T = W_1 + W_2 + W_3 + W_4...
\]

Depending on the design of the underground structure, the total vertical downward forces (\( W_T \)) may or may not be the same for all applications. In a conservative approach, the design of underground structures assumes that the water table, at the specific site, is at grade. With this being the case, it is essential that all vertical downward forces (\( W_T \)) be accounted for, to ensure that the structure will not float (\( W_T > F_B \)). For an underground structure, designed for a worst-case scenario, the following vertical downward forces (\( W \)) need to be accounted for:

- Weight of all walls and slabs
- Weight of soil on slabs
- Weight of soil on shelf/s
- Weight of equipment inside structure (permanent)
- Weight of inverts inside structure
- Friction of soil to soil
- Additional concrete added inside structure
- Weight of reinforcing steel

As noted earlier, not all underground structures are the same, and therefore some of the listed vertical downward forces (\( W \)) above may not be included in the summation of total vertical downward force (\( W_T \)).

3.1 Weight of Concrete

The total weight of the concrete that makes up the structure can account for a large portion of the downward gravitational force. In the design of an underground precast concrete structure a certain thickness of the walls and slabs must be implemented, whereas other products, such as HDPE have thinner wall sections and tend to float up out of the ground when they are pumped (if not properly anchored). With a specific gravity of 2.40, precast concrete products resist the buoyant forces associated with underground construction. In comparison, fiberglass has a specific gravity of 1.86 and high-density polyethylene (HDPE) has a specific gravity of 0.97.
The weight of the walls and slabs can determined by calculating the volume \( V \), in \( \text{ft}^3 \) (\( \text{m}^3 \)), of the underground structure and multiplying the volume by the density \( \rho \), in \( \text{lbs/ft}^3 \) (\( \text{kg/m}^3 \)), of the concrete. The calculation for determining the weight of walls and slabs for a square and round manholes are listed below:

**RECTANGULAR**

\[
W_{\text{wall}} = l \times h \times t \times \rho
\]

- \( W_{\text{wall}} \) = weight of wall segment (lbs)
- \( l \) = length of wall segment (ft)
- \( h \) = width of wall segment (ft)
- \( t \) = thickness of wall segment (ft)
- \( \rho \) = density of concrete (lbs/ft\(^3\))

**ROUND**

\[
W_{\text{barrel}} = \pi (OD^2-ID^2)h \rho
\]

- \( W_{\text{barrel}} \) = weight of MH barrel section (lbs)
- \( OD \) = outside diameter of barrel (ft)
- \( ID \) = inside diameter of barrel (ft)
- \( h \) = inside height of section

\[
W_{\text{slab}} = l \times h \times t \times \rho
\]

- \( W_{\text{slab}} \) = weight of slab (lbs)
- \( l \) = length of slab segment (ft)
- \( h \) = width of slab segment (ft)
- \( t \) = thickness of slab segment (ft)
- \( \rho \) = density of concrete (lbs/ft\(^3\))

\[
W_{\text{slab bar}} = \pi d^2/4 t \rho
\]

- \( W_{\text{slab bar}} \) = weight of round slab (lbs)
- \( d \) = diameter of round slab (ft)

As a general guideline, the range of values for heavy, normal, and lightweight concrete densities are as follows:

- **Heavy**: 165-330 (lbs/ft\(^3\))
- **Normal**: 145-150 (lbs/ft\(^3\))
- **Lightweight**: 90-115 (lbs/ft\(^3\))

### 3.2 Subtract for Openings

When calculating the weight of an underground precast concrete structure, openings must be accounted for and subtracted from the walls and slabs of the structure. The following equations are used to determine the weight of the openings removed.

**Volume of Cylinder Opening**

\[
V_{\text{cylinder}} = \pi \times r^2 \times t
\]

- \( V_{\text{cylinder}} \) = volume of cylinder (ft\(^3\))
- \( \pi \) = 3.14 (appx.)
- \( r \) = radius of circular opening (ft)
- \( t \) = member thickness (ft)
Weight of Subtracted Circular Opening

\[ W_{\text{cylinder\_opening}} = v_{\text{cylinder}} \times \rho \]

\[ W_{\text{cylinder\_opening}} = \text{weight of circular cutout (lbs)} \]
\[ v_{\text{cylinder}} = \text{volume of circular cutout (ft}^3\text{)} \]
\[ \rho = \text{density of concrete (lbs/ft}^3\text{)} \]

Volume of a Cube Opening

\[ v_{\text{cube}} = l \times w \times t \]

\[ v_{\text{cube}} = \text{volume of cube (ft}^3\text{)} \]
\[ l = \text{length of cube (ft)} \]
\[ w = \text{width of cube (ft)} \]
\[ t = \text{member thickness (ft)} \]

Weight of Subtracted Rectangular Opening

\[ W_{\text{cube\_opening}} = v_{\text{cube}} \times \rho \]

\[ W_{\text{cube\_opening}} = \text{weight of rectangular cutout (lbs)} \]
\[ v_{\text{cube}} = \text{volume of cube cutout (ft}^3\text{)} \]
\[ \rho = \text{density of concrete (lbs/ft}^3\text{)} \]

3.3 Weight of Earth Fill

The weight of the soils acting vertically on the slab is determined by multiplying the surface area of the structure by the depth of fill and the density of the soil. The calculation for determining the weight of the soil on a square manhole is listed below:

**RECTANGULAR**

\[ W_{\text{soil}} = l \times w \times d \times \rho \]

\[ W_{\text{soil}} = \text{weight of earth fill (lbs)} \]
\[ l = \text{length of structure (ft)} \]
\[ w = \text{width of structure (ft)} \]
\[ d = \text{depth of structure from surface (ft)} \]
\[ \rho = \text{density of soil (lbs/ft}^3\text{)} \]

**ROUND**

\[ W_{\text{soil}} = \pi (OD^2-ID^2)h \times \rho \]

\[ W_{\text{soil}} = \text{weight of earth fill (lbs)} \]
\[ OD = \text{outside diameter of barrel (ft)} \]
\[ ID = \text{inside diameter of barrel (ft)} \]
\[ h = \text{inside height of section} \]
\[ \rho = \text{density of soil (lbs/ft}^3\text{)} \]
3.4 Weight of Overburden Soil on Extended Base

Determining the weight of the soils on a shelf (extended base) is the same as calculating weight of the soils on the slab. The weight of the soils on the shelf is determined by multiplying the surface area of the shelf by the depth of the structure and the density of the soil. The calculation for determining the weight of the soil on a square manhole is listed below:

\[ W_{shelf} = l \times w \times d \times \rho \]

- \( W_{shelf} \) = weight of earth on shelf (lbs)
- \( l \) = length of segment (ft)
- \( w \) = width of segment (ft)
- \( d \) = depth of shelf from surface (ft)
- \( \rho \) = density of soil (lbs/ft\(^3\))

Extending the bottom slab to create a shelf outside the walls of the underground structure adds resistance to the buoyant force (\( F_b \)). Additional weight is obtained from the supplementary soil above the shelf.

3.5 Frictional Resistance (Extended Bases)

Adding a shelf (extended base) to the bottom slab of an underground structure not only adds an additional vertical downward force to the structure it also adds some frictional resistance from the soil.

This is calculated by adding the buoyant weight of the soil wedge created by the base extension (see Figure 2 below)

---

**Figure 2:** Frictional Resistance

- \( W_1 \) - Weight of the soil on the lid
- \( W_2 \) - Weight of the concrete structure
- \( W_3 \) - Weight of the soil on the shelves
- \( W_4 \) - Weight of the soil wedge caused by soil friction
- \( \sigma' \) - Soil internal friction angle
- \( f_s \) - Buoyant force
Soil friction angles vary depending on the type of soil, and its cohesiveness.

Typical angles are as follows:

<table>
<thead>
<tr>
<th>SOIL TYPE</th>
<th>ANGLE $\phi$, DEGREES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand and gravel mixture</td>
<td>33 – 36</td>
</tr>
<tr>
<td>Well-graded sand</td>
<td>32 – 35</td>
</tr>
<tr>
<td>Fine to medium sand</td>
<td>29 – 32</td>
</tr>
<tr>
<td>Silty sand</td>
<td>27 – 32</td>
</tr>
<tr>
<td>Silt (non-plastic)</td>
<td>26 – 30</td>
</tr>
</tbody>
</table>

3.6 Bench Walls, Inverts, Etc.

If there happens to be a need for benches, inverts, etc. inside the structure, this adds weight ($W_{\text{invert}}$) to the structure, increasing its total vertical downward force ($W_T$). The weight of the bench, invert, etc. should be calculated and factored into the total downward vertical forces acting on the structure.

3.7 Equipment Weights (Permanent)

If there is equipment that is known to be a permanent fixture inside the structure, this adds weight ($W_{\text{equip}}$) to the structure, increasing its total vertical downward force ($W_T$). The weight of the equipment may be obtained by the equipment manufacturer and factored into the total downward vertical forces acting on the structure.
Section 4- Computing Upward Buoyant Force

4.1 Buoyant Force ($F_b$)

As stated in *Archimedes’ principle*, an object is buoyed up by a force equal to the weight of the fluid displaced. Mathematically the principle is defined by the equation:

$$F_b = \gamma_f \times v_d$$

- $F_b =$ buoyant force (lbs)
- $\gamma_f =$ density of the water (62.4 lbs/ft$^3$)
- $v_d =$ displaced volume of the fluid (ft$^3$)

When analyzing buoyancy related concrete applications, the application is typically below grade and stationary. Assuming the application is stationary in a fluid, analysis requires an application of the static equilibrium equation in the vertical direction, $\Sigma F_v = 0$. Analyzing buoyancy related to underground structures requires an application of the same static equilibrium equation, assuming the structure to be stationary and either submerged or partially submerged in a fluid (in this case the surrounding soil /fill materials and any associated ground water).
Section 5- Safety Factor

The factor of safety considers the relationship between a resisting force and a disturbing force. In this case, it’s the relationship between the weight of the structure, and the force of the uplift caused by buoyancy. Failure occurs when that factor of safety is less than 1.0.

5.1 Guide for selecting an Appropriate Factor of Safety

Generally speaking, the higher the factor of safety (FS) the greater the impact to project / structure. An optimal design would be an appropriate FS which does the job for the conditions present at that specific site. It is recommended that the designer choose an appropriate FS after obtaining reviewing information about the jobsite.

According to ACI 350, the safety factor against flotation is usually computed as the total dead weight of the structure divided by the total hydrostatic uplift force. The factor of safety (FS) should reflect the risk associated with hydrostatic loading conditions.

In situations of flooding to the top of the structure and using dead weight resistance only, a FS of 1.10 is commonly used. In flood zone areas, or where high ground water conditions exists, a FS of 1.25 can be used. Where maximum ground water or flood levels are not well defined, or where soil friction is included in the flotation resistance, higher FS values should be considered.

5.2 Computing the Factor of Safety (FS)

A factor of safety can be established from the following calculation:

$$\text{Factor of Safety (FS)} = \frac{\text{Down Forces}}{\text{Up Force}} = \frac{W_T}{F_b}$$

- $W_T > F_b$ Structure will remain stationary
- $W_T < F_b$ Structure will float or shift upwards

When FS is less than 1, the up force will be greater than the down forces, which means that the structure will float. When the FS is greater than 1, the up force will be less than the down forces, which means that the structure will not float.
Section 6- Buoyancy Countermeasures

There are several methods that can be used in the industry to overcome a buoyancy problem. If the design of the underground structure does not meet the required safety factor, there are ways to go about fixing the problem.

Here are a few of the different methods used to overcome buoyancy, both before and after shifting or floatation.

6.1 Base Extension (CIP or Precast)

Using the additional weight of soil by adding shelves is a common method used to counteract buoyancy. By extending the bottom slab horizontally, this creates a shelf outside the walls of the structure and adds additional resistance to the buoyant force. The additional vertical downward force comes from the additional weight of the soil acting on the shelves ($W_{shelf}$). The size of the shelf can be designed however large and wide as needed so the buoyant force is resisted. However, limits in shipping width must be considered. In many cases, this is the most cost effective method used to repel the buoyant force ($F_b$). When pouring the shelf in place, mechanical connections must be designed to resist the vertical shear forces. It is best to have the shelf monolithically poured with the structure if possible. Figure 3 demonstrates the additional force that is added when a shelf is included in the design.

![Figure 3: Base Extension](image-url)

*W1* - Weight of soil on lid
*W2* - Weight of concrete structure
*W3* - Weight of soil on shelf
6.2 Anti-Floatation Slab

Another method that has been used before in construction is anchoring the structure to a large concrete mass (shelf) poured on-site or using precast concrete brought in off-site. The structure will sit directly on top of this large concrete mass, which has been previously poured in place at the site or cast, cured, and brought in from an offsite manufacturer. This method can cause problems, however, because both the base slabs must sit flush on top of each other. If they are not aligned perfectly, then cracking due to point loads may result. Poured in place concrete can be expensive and cause delays due to waiting for the required strength to be reached. Precast concrete alleviates such delays and the setting time, but the sub-base must be correctly level and aligned in order for the 2 slabs to sit flush. A mortar bed between the two surfaces is recommended. (See Figure 4)

![Diagram of Anti-Floatation Slab]

**Figure 4:** Anti Floatation Slab

In order to design the mechanical connection between the anti-floatation slab and the structure, the net upward force must be calculated. This can be achieved by multiplying the buoyant force by the factor of safety, and subtracting the downward force.

**Connection force** = \( F_b \times FS - W_t \)
6.3 Increase Member Thickness

One method that is used to overcome buoyancy is to increase the concrete's mass ($m$). This is accomplished by increasing member thickness (i.e. walls and slabs). Increasing the thickness of the walls and slabs can add a significant downward gravitational force but this may not always be the best bang for your buck. This can be a costly alternative due to the increasing costs of materials and production.

6.4 Make Structure Deeper and Fill with Additional Concrete

An additional method used to overcome buoyancy, would be to make the precast structure deeper than required for its functional purposes. This will add additional soil weight on top of the structure to oppose buoyant forces. Also, with the structure being deeper in the soil some contractors opt to pour additional concrete into the base of the installed precast concrete structure. This will again add more mass to the structure, which helps overcome buoyancy ($W_t > F_b$).

6.5 Anchor Structure

Another method used to overcome buoyancy is to provide anchors to help tie down the structure. These anchors are typically independent of the structure as illustrated by the following drawing:
Section 7 - Recap

To review all the other sections above and determine a plan of action, these are the essential considerations, recommended by Gary Munkelt, in “Precasters Notebook”, that must be made when writing specifications about buoyant forces.

1. Use data obtained from either a soils report or other means described in the first part of Section II. Then from the information obtained, choose a reasonable water level in ground (WL) and a safety factor (FS), which are suitable for the given jobsite conditions. Try and produce an optimally designed structure, since designing an underground structure with a higher than needed WL and FS, will result in higher production cost as well as other underlying costs.

2. Selection of appropriate backfill material. This is needed when backfilling over and around the structure as well as the shelf as noted in section 3.5.

3. The precaster must develop a method to resist buoyancy and back the method with calculations. In those calculations, the precaster must show how the structure will resist the buoyant force and determine a FS. Developing a plan such as this will help the precaster provide the most cost effective product.

4. Submit calculations to the owner of the precast product for approval. These calculations need to include, the depth of the ground water level below grade, and state the FS and supply data and information to why the FS is such. Factors such as the weight of the structure, the weight of the soil over the structure and shelves, to name a few should be included.

It is essential when designing an underground precast structure that the contractor and the precast manufacturer work together. The precast manufacturer needs vital information which the contractor usually has access to, and these bits of information will help the precast manufacturer design for the most cost effective structure which prevents floatation or shifting. The most cost effective method is determined after considering the site conditions, manufacturing methods, and the material handling issues.
Section 8- Examples

8.1 Provide at least 4 examples showing how various countermeasures can be applied

**Problem:** A structure is to be installed underground in a location where the water table is at grade. Verify that the following structure will resist buoyancy using a factor of safety of 1.1.

- **Inside Length (L)**: 10 ft
- **Top Slab Thickness**: 8 in
- **Outside Length (OL)**: 11.3333 ft
- **Inside Width (W)**: 8 ft
- **Wall Thickness**: 8 in
- **Outside Width (OW)**: 9.3333 ft
- **Inside Height (H)**: 12 ft
- **Bottom Slab Thickness**: 8 in
- **Outside Height (OH)**: 13.3333 ft
- **Earth Fill (F)**: 1 ft
- **Water Table Depth**: 0 in
- **Unit Weight of Concrete (Wc)**: 150 lbs/ft$^3$
- **Top Slab Opening**: 24 in diameter
- **Unit Weight of Soil (Ws)**: 120 lbs/ft$^3$
- **Wall openings**: 36 in diameter
- **Unit Weight of Water (Ww)**: 62.4 lbs/ft$^3$
- **Safety Factor Required**: 1.1

![Plan View](image1)

![Section View](image2)
Step 1: Calculate downward forces

Concrete weight:
(\(OL \times OW \times OH - L \times W \times H\))x\(Wc\)

\((11.3333 \times 9.3333 \times 13.3333 - 10 \times 8 \times 12) \times 150 = 67556 \text{ lbs}\)

Weight of fill:
OLxOWxFxWs

\(11.3333 \times 9.3333 \times 1 \times 120 = 12693 \text{ lbs}\)

Top slab opening
\((\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc + (\pi \times (\text{dia}/2)^2 \times F) \times Ws\)

\((3.14159 \times (2/2)^2 \times .6666) \times 150 + (3.14159 \times (2/2)^2 \times 1) \times 120 = 691 \text{ lbs}\)

Wall openings
\([((\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc \times 2] - 691 \text{ lbs}\)

\([((3.14159 \times (3/2)^2 \times \text{wall thickness}) \times 150 \times 2] = 1414 \text{ lbs}\)

Total Downward Force (Wt):
\[
\begin{align*}
67556 \text{ lbs} \\
12693 \text{ lbs} \\
-691 \text{ lbs} \\
-1414 \text{ lbs} \\
\hline
78144 \text{ lbs}
\end{align*}
\]

Step 2: Calculate upward buoyant forces

\(Fb = OL \times OW \times (OH+F - WT) \times Ww\)

\(11.3333 \times 9.3333 \times (13.3333+1-0) \times 62.4 = 94608 \text{ lbs}\)

Step 3: Difference between downward force and upward force

\(Wt - Fb\)

\(78144 - 94608 = -16464 \text{ lbs (upward)}\)

Step 4: Safety Factor

\(FS = Wt/Fb\)

\(FS = 78144 / 94608 = 0.826 < 1.1 \text{ therefore structure will not resist buoyancy forces with FS of 1.1.}\)
There are 4 possible solutions:

1. Make the structure deeper and add concrete fill to the inside
2. Add weight by increasing member thicknesses
3. Add an extension to the outside of the base slab to engage the soil outside the structure
4. Add a separate anti-floatation slab below the base slab

**SOLUTION # 1 – Make the structure deeper and add concrete fill to the inside**

**Step 1 : Calculate the required additional weight**

Upward buoyant force \( F_b = 94,608 \text{ lbs} \times \text{FS (1.1)} = 104,068 \text{ lbs} \)

Downward Force \( W_t = 78,144 \text{ lbs} \)

Additional weight required = \( 104,068 - 78,144 = 25,924 \text{ lbs} \)

**Step 2 : Calculate additional depth required**

The buoyant weight of concrete is \( 150 \text{ lbs/ft}^3 - 62.4 \text{ lbs/ft}^3 = 87.6 \text{ lbs/ft}^3 \)

Volume of concrete required: \( 25,924 \text{ lbs} / 87.6 \text{ lbs/ft}^3 = 295.9 \text{ ft}^3 \) (10.96 yd^3)

Current outside area : \( 11.3333 \text{ ft} \times 9.3333 \text{ ft} = 105.8 \text{ ft}^2 \)

Additional depth required: \( 295.9 \text{ ft}^3 / 105.8 \text{ ft}^2 = 2.798 \text{ ft} \) – Add 3’ to inside height

**Step 3 : Recalculate FS with additional 3’ of concrete**

<table>
<thead>
<tr>
<th>Inside Length (L)</th>
<th>10 ft</th>
<th>Top Slab Thickness</th>
<th>8 in</th>
<th>Outside Length (OL)</th>
<th>11.3333 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside Width (W)</td>
<td>8 ft</td>
<td>Wall Thickness</td>
<td>8 in</td>
<td>Outside Width (OW)</td>
<td>9.3333 ft</td>
</tr>
<tr>
<td>Inside Height (H)</td>
<td>15 ft</td>
<td>Bottom Slab Thickness</td>
<td>8 in</td>
<td>Outside Height (OH)</td>
<td>16.3333 ft</td>
</tr>
<tr>
<td>Earth Fill (F)</td>
<td>1 ft</td>
<td>Water Table Depth</td>
<td>0 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Concrete (Wc)</td>
<td>150 lbs/ft^3</td>
<td>Top Slab Opening</td>
<td>24 in diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Soil (Ws)</td>
<td>120 lbs/ft^3</td>
<td>Wall openings</td>
<td>36 in diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Water (Ww)</td>
<td>62.4 lbs/ft^3</td>
<td>Safety Factor Required</td>
<td>1.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Concrete weight:
\[(OL\times OW\times OH - L\times W\times H)\times Wc\]
\[(11.333\times 9.333\times 16.333 - 10 \times 8 \times 15) \times 150 = 79156 \text{ lbs}\]

Weight of fill:
\[OL\times OW\times FxW\]
\[11.333\times 9.333\times 1\times 120 = 12693 \text{ lbs}\]

Weight of Infill:
\[L\times W\times \text{Height of infill}\times Wc\]
\[10 \times 8 \times 3 \times 150 = 36000 \text{ lbs}\]
Top slab opening
\( (\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc + (\pi \times (\text{dia}/2)^2 \times F) \times Ws \)

\( (3.14159 \times (2/2)^2 \times 0.667) \times 150 + (3.14159 \times (2/2)^2 \times 1) \times 120 = 691 \text{ lbs} \)

Wall openings
\( [(\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc]^2 \)

\( [(3.14159 \times (3/2)^2 \times \text{wall thickness}) \times 150]^2 = 1414 \text{ lbs} \)

Total Downward Force (Wt)=

\[ 79152 \text{ lbs} \]
\[ 12693 \text{ lbs} \]
\[ 36000 \text{ lbs} \]
\[ -691 \text{ lbs} \]
\[ -1414 \text{ lbs} \]
\[ 125744 \text{ lbs} \]

**Step 2: Calculate upward buoyant forces**

\( Fb = OL \times OW \times (OH+F-WT) \times Ww \)

\( 11.3333 \times 9.3333 \times (16.3333+1-0) \times 62.4 = 114409 \text{ lbs} \)

**Step 3 : Calculate FS**

\( FS = \frac{Wt}{Fb} \)

\( FS = 125744 / 114409 = 1.1 \text{ OK} \)

**SOLUTION #2 – Add to member thicknesses**

Let’s assume a 12” top slab, 13” walls and 12” bottom slab

- **Inside Length (L)**: 10 ft
- **Top Slab Thickness**: 12 in
- **Outside Length (OL)**: 12.1666 ft
- **Inside Height (H)**: 12 ft
- **Bottom Slab Thickness**: 12 in
- **Outside Height (OH)**: 14 ft
- **Earth Fill (F)**: 1 ft
- **Water Table Depth**: 0 in
- **Unit Weight of Concrete (Wc)**: 150 lbs/ft\(^3\)
- **Unit Weight of Soil (Ws)**: 120 lbs/ft\(^3\)
- **Unit Weight of Water (Ww)**: 62.4 lbs/ft\(^3\)
- **Top Slab Opening**: 24 in diameter
- **Wall openings**: 36 in diameter
- **Safety Factor Required**: 1.1
Step 1: Calculate downward forces

Concrete weight:
\[(OL \times OW \times OH - L \times W \times H) \times Wc\]
\[(12.1666 \times 10.1666 \times 14 - 10 \times 8 \times 12) \times 150 = 115758 lbs\]

Weight of fill:
\[OL \times OW \times F \times Ws\]
\[12.1666 \times 10.1666 \times 1 \times 120 = 14843 lbs\]

Top slab opening
\[(\pi \times (dia/2)^2 \times \text{slab thickness}) \times Wc + (\pi \times (dia/2)^2 \times F) \times Ws\]
\[(3.14159 \times (2/2)^2 \times 1) \times 150 + (3.14159 \times (2/2)^2 \times 1) \times 120 = 848 lbs\]

Wall openings
\[[(\pi \times (dia/2)^2 \times \text{wall thickness}) \times Wc] \times 2\]
\[[(3.14159 \times \frac{3}{2})^2 \times 1.07] \times 150 \times 2 = 2297 \text{ lbs}\]

Total Downward Force (Wt) =
\[
\begin{align*}
115758 \text{ lbs} \\
14843 \text{ lbs} \\
-848 \text{ lbs} \\
-2297 \text{ lbs} \\
\textbf{127 456 lbs}
\end{align*}
\]

**Step 2: Calculate upward buoyant forces**

\[
F_b = OL \times OW \times (OH + F - WT) \times W_w
\]

\[
12.1666 \times 10.1666 \times (14 + 1 - 0) \times 62.4 = 115 778 \text{ lbs}
\]

**Step 3: Safety Factor**

\[
FS = \frac{W_t}{F_b}
\]

\[
FS = \frac{127 456}{115 778} = 1.1 \text{ OK}
\]

**SOLUTION #3 – Add an extension to the outside of the base slab to engage the soil outside the structure**

<table>
<thead>
<tr>
<th>Inside Length (L)</th>
<th>10 ft</th>
<th>Top Slab Thickness</th>
<th>8 in</th>
<th>Outside Length (OL)</th>
<th>11.3333 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside Width (W)</td>
<td>8 ft</td>
<td>Wall Thickness</td>
<td>8 in</td>
<td>Outside Width (OW)</td>
<td>9.3333 ft</td>
</tr>
<tr>
<td>Inside Height (H)</td>
<td>12 ft</td>
<td>Bottom Slab Thickness</td>
<td>8 in</td>
<td>Outside Height (OH)</td>
<td>13.3333 ft</td>
</tr>
<tr>
<td>Earth Fill (F)</td>
<td>1 ft</td>
<td>Water Table Depth</td>
<td>0 in</td>
<td>Base Extension</td>
<td>6 in</td>
</tr>
<tr>
<td>Unit Weight of Concrete (Wc)</td>
<td>150 lbs/ft(^3)</td>
<td>Top Slab Opening</td>
<td>24 in diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Soil (Ws)</td>
<td>120 lbs/ft(^3)</td>
<td>Wall openings</td>
<td>36 in diameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Water (Ww)</td>
<td>62.4 lbs/ft(^3)</td>
<td>Safety Factor Required</td>
<td>1.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Step 1: Calculate downward forces

Concrete weight:
\[(OL \times OW \times OH - L \times W \times H) \times Wc\]
\[(11.333 \times 9.333 \times 13.333 - 10 \times 8 \times 12) \times 150 = 67556 \text{ lbs}\]

Weight of fill:
\[OL \times OW \times F \times Ws\]
\[11.333 \times 9.333 \times 1 \times 120 = 12693 \text{ lbs}\]

Buoyant weight of soil engaged by extension
Perimeter of box * shelf width * soil height * (Ws-Ww)
\[(11.333 \text{ ft} + .5 \text{ ft} + 9.333 \text{ ft} + .5 \text{ ft} + 11.333 \text{ ft} + .5 \text{ ft} + 9.333 \text{ ft} + .5 \text{ ft}) \times .5 \text{ ft} \times 13.666 \times (120-62.4)\]
\[= 17055 \text{ lbs}\]

Top slab opening
\[(\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc + (\pi \times (\text{dia}/2)^2 \times F) \times Ws\]
\[(3.14159 \times (2/2)^2 \times .667) \times 150 + (3.14159 \times (2/2)^2 \times 1) \times 120 = 691 \text{ lbs}\]

Wall openings
\[[(\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times Wc]\times 2\]
\[(3.14159 \times (3/2)^2 \times \text{wall thickness}) \times 150 \times 2 = 1414 \text{ lbs}\]

**Total Downward Force (Wt)=**

- 67556 lbs
- 12693 lbs
- 17055 lbs
- -691 lbs
- -1414 lbs
- **95 199 lbs**

**Step 2: Calculate upward buoyant forces**

\[F_b = OL \times OW \times (OH + F - WT) \times Ww\]

\[11.3333 \times 9.3333 \times (13.3333 + 1 - 0) \times 62.4 = 94 608 \text{ lbs}\]

**Step 4 : Safety Factor**

\[FS = \frac{W_t}{F_b}\]

\[FS = \frac{95 199}{94 608} = 1.0 < 1.1 \text{ which doesn’t work.}\]

Since the factor of safety without considering a soil wedge is greater than 1.0, then compute the weight of soil within the soil wedge.

Assume a friction of angle \(\sigma\) of 10 degrees (conservative)

**Step 5: Calculate additional downward force due to soil wedge**

Through basic trigonometry, we find that with a 10 degree angle, our soil wedge is 2.41 ft wide at the top of the soil. We must calculate that wedge volume over the entire perimeter of the structure.
Outside perimeter of base extension = 12.3333*2 + 10.3333*2 = 45.3333 ft.

Volume = ((2.41 * 13.6667) /2)*45.3333 = 746.69 ft³

Weight = 749.69 * (120 lbs/ft³ – 62.4 lbs/ft³) = 42 999 lbs

Add to downward force of

95 199 lbs
138 198 lbs

Step 6: Calculate new FS

FS = Wt/Fb

FS = 138 198 / 94 608 = 1.46 > 1.1  OK

SOLUTION #4 – Add a separate anti-floating slab below the base slab

Inside Length (L)  10 ft  Top Slab Thickness  8 in  Outside Length (OL)  11.3333 ft
Inside Width (W)  8 ft  Wall Thickness  8 in  Outside Width (OW)  9.3333 ft
Inside Height (H)  12 ft  Bottom Slab Thickness  8 in  Outside Height (OH)  13.3333 ft
Earth Fill (F)  1 ft  Water Table Depth  0 in  Base Extension  6 in
Unit Weight of Concrete (Wc)  150 lbs/ft³  Top Slab Opening  24 in diameter
Unit Weight of Soil (Ws)  120 lbs/ft³  Wall openings  36 in diameter
Unit Weight of Water (Ww)  62.4 lbs/ft³  Safety Factor Required  1.1
Step 1: Calculate downward forces

Concrete weight:
\[(\text{OL} \times \text{OW} \times \text{OH} - \text{L} \times \text{W} \times \text{H}) \times \text{Wc}\]

\[(11.333 \times 9.333 \times 13.333 - 10 \times 8 \times 12) \times 150 = 67556 \text{ lbs}\]

Weight of fill:
\[\text{OL} \times \text{OW} \times \text{FxWs}\]

\[11.333 \times 9.333 \times 1 \times 120 = 12693 \text{ lbs}\]

Buoyant weight of soil engaged by extension

Perimeter of box * shelf width * soil height * (Ws-Ww)

\[(11.333 \text{ ft} + .5 \text{ ft} + 9.333 \text{ ft} + .5 \text{ ft} + 11.333 \text{ ft} + .5 \text{ ft} + 9.333 \text{ ft} + .5 \text{ ft}) \times .5 \text{ ft} \times 13.6666 \times (120 - 62.4) = 17055 \text{ lbs}\]

Top slab opening

\[(\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times \text{Wc} + (\pi \times (\text{dia}/2)^2 \times F) \times \text{Ws}\]

\[(3.14159 \times (2/2)^2 \times .667) \times 150 + (3.14159 \times (2/2)^2 \times 1) \times 120 = 691 \text{ lbs}\]

Wall openings

\[\left\{ (\pi \times (\text{dia}/2)^2 \times \text{slab thickness}) \times \text{Wc}\right\}^2\]

\[\left\{ (3.14159 \times (3/2)^2 \times \text{wall thickness} \times 150\right\}^2 = 1414 \text{ lbs}\]
Total Downward Force (Wt) = 
67556 lbs
12693 lbs
17055 lbs
-691 lbs
-1414 lbs
95199 lbs

Step 2: Calculate upward buoyant forces

\[ F_b = OL \times OW \times (OH - F - WT) \times Ww \]

11.3333 * 9.3333 * (13.3333 + 1 - 0) * 62.4 = 94 608 lbs

Step 4 : Safety Factor

FS = Wt / Fb

FS = 95 199 / 94 608 = 1.0 < 1.1 which doesn’t work.

Add an anti-floatation slab that is 6” bigger than the structure on all sides.

Step 5 : Calculate the additional downward force created by the slab

Slab volume: 12.3333 ft * 10.3333 ft * 1 ft = 127.4 ft³

Weight : 127.4 * (Wc - Ww) = 127.4 * (150 - 62.4) = 11 164 lbs

Add to downward force

95 199 lbs
106 363 lbs

Step 6: Calculate new FS

FS = Wt / Fb

FS = 106 363 / 94 608 = 1.12 > 1.1 OK

Step 7 : Calculate connection force for mechanical connection between slab and structure

Connection force = (Fb * FS) – Wt = (94608 * 1.1) – 95199 = 8870 lbs